Theoretical Analyses of Lightpath Blocking Performance in CO-OFDM Optical Networks with/without Spectrum Conversion

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***Abstract*—This paper provides the first analysis of the impact of spectrum conversion on the lightpath blocking performance in the CO-OFDM optical networks. Two analytical models for the cases of using and not using spectrum convertors are developed for an *H*-hop end-to-end lightpath and a network, respectively. Numerical results show that the use of spectrum convertors can significantly improve the lightpath blocking performance.**

***Index Terms*—CO-OFDM, spectrum assignment, spectrum continuity, spectrum conversion, lightpath blocking.**

I. Introduction

A NOVEL optical transmission technology, called Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) [1], has emerged as an attractive and promising solution for the future optical transport networks in tackling the disadvantage of low spectrum efficiency in traditional WDM networks. As one of the major advantages, the CO-OFDM technology is not subject to the constraint of the fixed frequency grids, and shows great potential in improving spectrum utilization and flexibility in provisioning variable network bandwidth. Lots of research has applied the CO-OFDM technology to the optical transport network. A novel network configuration called Spectrum-Slice Elastic Optical Path (SLICE) Network was proposed in [2]. This bandwidth-elastic network configuration can provision capacity for an optical channel at arbitrary data-rates, ranged from sub-wavelength to super-wavelength granularities. Various schemes on routing and spectrum assignment (RSA) were developed for the SLICE networks [3]. The concept of bandwidth squeezing used for network recovery was proposed in [4]. Finally, Shen et al. [5] compared two spectrum operational modes, namely mini-grid and gridless to find that a network operated under the mini-grid model can achieve almost the same good performance as that of the gridless operational mode.

However, most of the existing studies on the bandwidth-elastic optical networks have been performed under the assumption of continuous lightpath spectrum. Very few works have considered the benefit of using spectrum convertors as that of using wavelength convertors in the traditional WDM networks [6, 7]. In a bandwidth-elastic optical transport network, the use of spectrum convertors is expected to show similar benefits in improving network performance, especially when network resources (i.e., channel spectrum) are divided into much finer granularities, which makes the allocation even more flexible. However, to the best of our knowledge, the benefit of spectrum conversion has not yet been well-explored for the bandwidth-elastic CO-OFDM optical networks.

Spectrum conversion can be realized by O/E/O process as described in [8]. The functionality of spectrum conversion may increase the complexity of the CO-OFDM node architecture and introduce an additional cost. However, there exists a great potential for significantly improving the network performance. This paper focuses on numerically evaluating how the capability of spectrum conversion can affect the performance of dynamic lightpath service provisioning in the CO-OFDM networks. Note that spectrum conversions always happen on continuous spectrum slots that are assigned to lightpath requests whenever needed. The current research problem is more challenging than the one in the traditional wavelength-routed networks. For instance, to successfully establish a lightpath that traverses H hops and requires S spectrum slots, only one free wavelength channel is needed on each of the H hops in the traditional wavelength-routed optical networks, as long as the wavelength bandwidth is larger than S spectrum slots. In contrast, in the CO-OFDM networks, to establish such a lightpath, $S(>1)$ continuous free spectrum slots must be found on all the H hops. Moreover, if there is no spectrum conversion capability within the network, the same S free continuous slots must be guaranteed on all the H links, making the spectrum assignment more complicated. Therefore, we develop analytical models to evaluate the lightpath provisioning performance for the cases of with and without spectrum conversion. The models can provide a valid evaluation on the benefit of using spectrum convertors in the bandwidth-elastic CO-OFDM networks.

II. Analytical Models for an H-hop Lightpath

In this section, an analytical model for an H-hop end-to-end lightpath is presented. Two cases including with and without spectrum conversion are considered and compared. In the former case, each node is equipped with spectrum convertors and is able to convert the spectrum of a lightpath to any other spectrum. In the latter case, nodes have no spectrum conversion capability and each provisioned lightpath has to obey the constraint of spectrum continuity.

We first illustrate three important concepts: Spectrum-slot Chain (SC), Spectrum Band (SB), and Spectrum-slot Chain Band (SCB). A SC is a chain of spectrum slots that span several continuous links, on which each has a spectrum slot at the same central frequency/color; a SB is a band that contains

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several consecutive spectrum slots on a single fiber link; a SCB is a two-dimensional concept, defined to be a band of continuous spectrum-slot chains.

A. Analytical models for an H-hop lightpath

On each of the links of the *H*-hop end-to-end lightpath, we assume there are a total of *F* spectrum slots. In addition, the number of required spectrum slots per lightpath request is assumed to be *S* and the idle probability of each spectrum slot is assumed to be given as *p*.

The blocking probability of a lightpath is co-determined by the occupation status (i.e., busy or idle) of spectrum slots on each link, and the number of required spectrum slots by the lightpath. For the analytical models, we first define the following notations: 1) P_{SC}: denotes the idle probability of a Spectrum-slot Chain (SC); 2) $P_{SB}(S,F)$: denotes the idle probability of an *S*-spectrum slot (*S*-slot in short) Spectrum Band (SB) on an *F*-slot link; 3) $P_{SCB}(S,F)$: denotes the idle probability of an *S*-slot Spectrum-slot Chain Band (SCB) when the total number of spectrum slots on each link is *F*; 4) $P_{LP}(S)$: denotes the blocking probability of an *S*-slot lightpath. Based on the above assumptions and notations, we present the analytical models with and without spectrum conversion.

For the case of *with spectrum conversion*, to successfully establish a lightpath, we need to ensure each of the fiber links traversed by the lightpath to have at least *S* continuous spectrum slots. Thus, the blocking probability of a lightpath request, i.e., $P_{LP}(S)$, can be calculated as

$$P_{LP}(S) = 1 - \prod\nolimits_{H:l\in H} P_{SB}(S,F) = 1 - P_{SB}(S,F)^H \quad (1)$$

where $H : l \in H$ means that the hop traversed by link *l* of the lighpath and $\prod_{H:l\in H} P_{SB}(S,F)$ means that a free SB with *S* spectrum slots can be found on each of the *H* hops.

The probability that a free *S*-slot SB can be found on a link with a total of *F* spectrum slots, i.e., $P_{SB}(S,F)$, is a function of the total number of spectrum slots *F*, the required number of continuous spectrum slots *S*, and the idle probability of each spectrum slot *p*. If we consider the idle-or-occupied status of a spectrum slot as the head-or-tail status of a coin and the total number of spectrum slots as the total number of tosses, then the probability of finding at least *S* continuous idle spectrum slots out of a total of *F* slots is equivalent to the probability of obtaining at least *S* continuous heads (or tails) among an *F*-time tossing event.

For the above tossing problem, the probability of obtaining at least *S* continuous heads (or tails) among a total of *F*-time tossing event, denoted as $P_{SB}(S,F)$, can be calculated as (2) [9]

$$P_{SB}(S,F) = \sum\nolimits_{j=1}^{S} P_{SB}(S,F-j)p^{j-1}(1-p) + p^s \quad (2)$$

where *p* is the probability of tossing a head (or tail). We can use equation (2) to compute the probability of finding at least *S* continuous idle spectrum slots out of the *F* spectrum slots on a link. In addition, $P_{SB}(S,F-j)$ is a iterative term to compute the probability of obtaining at least *S* continuous heads (or tails) among a (*F-j*)-time tossing event, which is equivalent to the probability of finding at least *S* continuous free slots out of a total of *F-j* free slots ($F-j \geq S$).

For the case of *without spectrum conversion*, we can extend (2) from a link to an end-to-end path. Specifically, we first find the idle probability of a specific SC, and then apply (2) by treating an SC along a path as a spectrum slot on a link. Given the idle probability of a spectrum slot on a link as *p*, the probability that a spectrum-slot chain is available is

$$P_{SC} = \prod\nolimits_{H} p = p^H \quad (3)$$

Next, if the term *p* in (2) is substituted with P_{SC} in (3), we can obtain the probability that *S* identical continuous spectrum slots are available on each of the links along an *H*-hop lightpath as (4). Finally, the blocking probability of an *S*-slot *H*-hop lightpath is given by (5).

$$P_{SCB}(S,F) = \sum\nolimits_{j=1}^{S} P_{SCB}(S,F-j)P_{SC}^{j-1}(1-P_{SC}) + P_{SC}^{S} \quad (4)$$

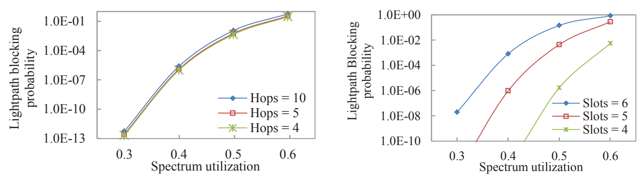
$$P_{LP}(S) = 1 - P_{SCB}(S,F) \quad (5)$$

B. Preliminary numerical results for an H-hop lightpath

Assume the spectrum slot utilization is p_b (equivalently, the idle probability of a spectrum slot is $p = 1 - p_b$), and each fiber link consists of 400 5GHz spectrum slots. The impacts of different numbers of hops and required spectrum slots per lightpath on the lightpath blocking performance are evaluated for the above two *H*-hop analytical models.

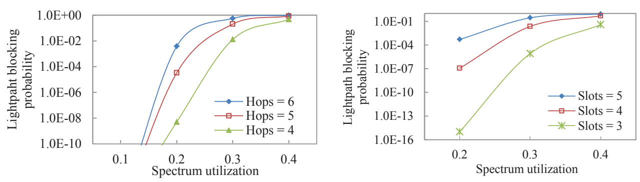
Under the condition of full spectrum conversion, Fig. 1(a) shows the impact of the number of lightpath hops on the lightpath blocking performance. The number of spectrum slots per lightpath connection is 5, i.e., *S*=5. The x-axis shows the spectrum slot utilization p_b. It is interesting to see that under different numbers of lightpath hops, i.e., 4, 5 and 10, the lightpath blocking probabilities are very close. This implies that with the full spectrum conversion capability, the lightpath blocking performance is not sensitive to the number of hops traversed by a lightpath. In Fig. 1(b), for a 4-hop lightpath, when the number of spectrum slots per lightpath increases from 4 to 6, it is interesting to see that the lightpath blocking performance is changing dynamically. This is reasonable since a larger number of spectrum slots means that more network resources are required by each lightpath.

Under the condition of no spectrum conversion capability, Fig. 2(a) shows the effect of different number of lightpath hops on the lightpath blocking performance. The number of required spectrum slots is set to be 5 for each lightpath and the numbers of hops change from 4 to 6. Different from the case of full spectrum conversion, we see that the number of lightpath hops demonstrates a much stronger impact on the lightpath blocking performance. This is attributed to the constraint of spectrum continuity, which requires all the fiber links traversed by a lightpath to use the same spectrum slots. It is generally more difficult for a longer lightpath to satisfy such a requirement. Similar to the case of with full spectrum conversion, for a 3-hop lightpath, with an increasing number of spectrum slots from 3 to 5 in Fig. 2(b), the lightpath blocking probability increases dramatically.



(a) (b)

Fig. 1. (a) Lightpath blocking probability under different lightpath hops (with spectrum conversion). Number of spectrum slots per lightpath connection = 5; (b) Lightpath blocking probability under different numbers of required spectrum slots (with spectrum conversion). Number of lightpath hops = 4;



(a) (b)

Fig. 2. (a). Lightpath blocking probability under different lightpath hops (without spectrum conversion). Number of spectrum slots per lightpath connection = 3; (b). Lightpath blocking probability under different number of required slots (without spectrum conversion). Number of lightpath hops = 3.

III. Analytical Models for a Network

To present the analytical model of a network, the following additional notations are required: 1)$P_{SB}^{l}(S,F)$: denotes the idle probability of an *S*-slot SB on link 1 with a total number of *F* spectrum slots; 2) P_{SC}^{R}: denotes the idle probability of a SC on route *R*; 3) $P_{SCB}^{R}(S,F)$: denotes the probability that an *S*-slot SCB is available on route *R* when the number of total spectrum slots on each link is *F*; 4) $P_R(S)$: denotes the blocking probability of *S*-slot lightpath on route *R*; 5) $L_R(S)$: denotes the offered *S*-slot lightpath traffic load on route *R*; 6) $P_N(S)$: denotes the average blocking probability of *S*-slot lightpath in the whole network.

A. Analytical model for CO-OFDM network with spectrum conversion

In the previous model for an *H*-hop end-to-end lightpath, the spectrum slot utilization is assumed to be given. For the network analytical models, this parameter is derived from other given parameters, including 1) a network physical topology; 2) a traffic load matrix which reflects the traffic load (in Erlangs) between each node pair (based on which $L_R(S)$ can be derived); and 3) the total number of spectrum slots on each fiber. In addition, different from the scenario of a single lightpath, as the traffic demands exist on different node pairs in the network, the spectrum resources on each fiber link can be contended by different lightpath demands from different node pairs. Thus, modeling the lightpath blocking performance under the network environment is much more complex. The model for a single *H*-hop lightpath can however function as an intermediate step in calculating the network lightpath blocking probability.

Again, assume the total number of spectrum slots on each fiber link is *F* and the required number of slots per lightpath request is *S*. Since the total traffic carried by the established lightpaths does not exceed the total link capacity, the spectrum slot utilization probability p_s^l and spectrum slot idle probability p_l on link *l* are calculated as

$$p_s^l = \frac{\sum_{R:l\in R} L_R(S)(1-P_R(S))}{F} \tag{6}$$

$$p_l = 1 - p_s^l = 1 - \frac{\sum_{R:l\in R} L_R(S)(1-P_R(S))}{F} \tag{7}$$

where *R*, $R : l \in R$, is the set of routes that traverses link *l*. As $L_R(S)$ is the offered load on route *R*, the term $L_R(S)(1-P_R(S))$ calculates the carried load on route *R* and $\sum_{R:l\in R} L_R(S)(1-P_R(S))$ calculates the total traffic load carried on link *l*, which can be considered as occupied spectrum resources on the link by all the routes traversing it. Thus, the term $\frac{\sum_{R:l\in R} L_R(S)(1-P_R(S))}{F}$ calculates the average carried load on each spectrum slot, which is equivalent to the average spectrum slot utilization.

Given the idle probability of each spectrum slot computed by (7), the calculation of *S*-slot SB availability on link *l*, i.e., $P_{SB}^{l}(S,F)$, is the same as (2) by replacing *p* with p_l, and is given by (8). Thus, the lightpath blocking probability on route *R* is given by (9).

$$P_{SB}^{l}(S,F) = \sum_{j=1}^{S} P_{SB}^{l}(S,F-j)p_l^{j-1}(1-p_l) + p_l^s \tag{8}$$

$$P_R(S) = 1 - \prod_{l:l\in R} P_{SB}^{l}(S,F) \tag{9}$$

where $\prod_{l:l\in R} P_{SB}^{l}(S,F)$ is the probability that a free *S*-slot SB can be found on each of the links along route *R*. Finally, the network average lightpath blocking probability is

$$P_N(S) = \frac{\sum_R L_R(S)P_R(S)}{\sum_R L_R(S)} \tag{10}$$

An iterative process is applied to calculate the above equations and find a final network lightpath blocking probability. We use a relaxation method [6] for the iterative calculation. The steps of the iterative process are as follows:

Step 1. Assign a random initial value between 0 and 1.0 to $P_R(S)^{(0)}$ and use it to calculate (6) and (7) to obtain $p_l^{(0)}$, where the superscript of each term is the index of iteration.

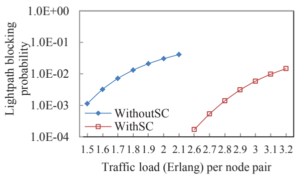
Step 2. According to $p_l^{(0)}$, calculate (8), (9), (10) to get $P_{SB}^{l}(S,F)^{(0)}$, $P_R(S)^{(1)}$, $P_N(S)^{(1)}$.

Step 3. Start the iterative process and use the obtained $P_R(S)^{(1)}$ to calculate (6), (7), and then (8), (9), (10) for *n* times to obtain $p_l^{(n-1)}$, and then obtain $P_{SB}^{l}(S,F)^{(n-1)}$, $P_R(S)^{(n)}$, $P_N(S)^{(n)}$.

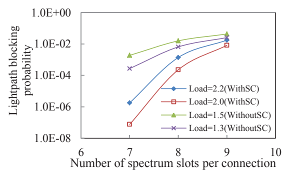
Step 4. If the difference between $P_N(S)^{(n)}$ and $P_N(S)^{(n-1)}$ is smaller than a pre-defined small threshold value, the iterative process stops, which means that the network blocking probability converges to a stable value. Otherwise, increase the iteration index *n* by one and go back to Step 3 to continue the iterative process.

B. Analytical model for CO-OFDM network without spectrum conversion

In this section, we consider the theoretical model for a network without spectrum conversion capability. The spectrum slot utilization probability p_s^l and the spectrum slot idle probability p_l on link *l* are calculated by (6) and (7), respectively. Based on the found spectrum slot idle probability p_l and under



(a)



(b)

Fig. 3. (a) Lightpath blocking probability under different lightpath hops (with spectrum conversion). Number of spectrum slots per lightpath connection = 5; (b) Lightpath blocking probability under different numbers of required spectrum slots (with spectrum conversion). Number of lightpath hops = 4;

the constraint of spectrum continuity, we can calculate the idle probability of a SC on route *R* as (11). According to (4), we can further find the probability that an *S*-slot SCB is available on route *R*, i.e., $P^R_{SCB}(S,F)$ as (12). The lightpath blocking probability on route *R* is given by (13).

$$P^R_{SC} = \prod\nolimits_{l:l\in R} p_l \tag{11}$$

$$P^R_{SCB}(S,F) = \sum\nolimits_{j=1}^{S} P^R_{SCB}(S,F-j) {P^R_{SC}}^{j-1} (1-P^R_{SC}) + {P^R_{SC}}^{S} \tag{12}$$

$$P_R(S) = 1 - P^R_{SCB}(S,F) \tag{13}$$

Finally, the network average lightpath blocking probability is calculated by (10), and a similar iterative calculation process as previously is applied to find the network blocking probability for the case of without spectrum conversion.

Note that the above two models are approximate, as we assumed that the blocking probability of each spectrum slot is the same. A more accurate assumption is that on each link, the spectrum slots located in the middle of the SB may have the same blocking probability, but the spectrum slots near the two ends of the SB may have a lower utilization. For this and other reasons, the results obtained from these approximate models may not match well with those from simulations. Nevertheless, similar to the rationale for developing an approximate model in [7], we believe approximate analytical models are not only unavoidable but also still useful as a tool to evaluate the benefits of spectrum convertors.

C. Numerical Results

Two types of CO-OFDM networks, namely with and without spectrum conversion capabilities, are compared in the terms of lightpath blocking probabilities. Assume there are 400 5GHz spectrum slots on each fiber link, and for each test point, the required number of spectrum slots by each lightpath request is the same. We employed the 14-node 21-link NSF network [6] as our test network and Dijkstras algorithm to find a fixed shortest route for each node pair.

Fig. 3(a) compares the cases of with and without spectrum conversion under different lightpath traffic demands when the number of required spectrum slots per lightpath is 5. It can be seen that the network performance for the case with full spectrum conversion (i.e., WithSC) is much better than that of the case of without spectrum conversion (i.e., WithoutSC). Under the same lightpath blocking probability saying 1%, the network with spectrum conversion can carry about 3.1 Erlang traffic load, while the case of without spectrum conversion can carry only about 1.75 Erlang traffic load. From the network perspective, this verifies that using spectrum convertors can significantly improve the network spectrum utilization and lightpath service provisioning performance. Fig. 3(b) shows the network lightpath blocking probabilities versus the number of required spectrum slots. Again, it can be seen that using spectrum convertors can significantly improve network lightpath blocking performance. Specifically, when the number of spectrum slots is the same, the lightpath blocking probability of the case with spectrum conversion under even high traffic load (i.e., when load = 2.2 and 2.0) is almost much lower than that of the case without spectrum conversion under much lower traffic load (i.e., when load = 1.5 and 1.3).

IV. Conclusion

In this paper, we have evaluated the benefit of spectrum conversion in the context of a bandwidth-elastic CO-OFDM optical transport network. Two analytical models have been developed to provide quick estimates of the lightpath blocking performance for the cases of using and not using spectrum convertors, respectively. The numerical results obtained from the models have shown that the use of spectrum convertors can significantly improve lightpath blocking performance and network spectrum resource utilization.

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References

[1] W. Shieh, *et al.*, "Coherent optical OFDM: has its time come?" *J. Optical Networking*, vol. 7, no. 3, pp. 234–255.
[2] M. Jinno, *et al.*, "Spectrum-efficient and scalable elastic optical path network: architecture, benefits, and enabling technologies," *IEEE Commun. Mag.*, vol. 47, no. 11, pp. 66–73, Nov. 2009.
[3] M. Jinno, *et al.*, "Distance-adaptive spectrum resource allocation in spectrum-sliced elastic optical path network," *IEEE Commun. Mag.*, vol. 48, no. 8, pp. 138–145, Aug. 2010.
[4] Y. Sone, *et al.*, "Highly survivable restoration scheme employing optical bandwidth squeezing in spectrum-sliced elastic optical path (SLICE) network," *2009 OFC/NFOEC.*
[5] G. Shen, *et al.*, "From coarse grid to mini-grid to gridless: how much can gridless help contentionless?" *2011 OFC/NFOEC.*
[6] G. Shen, *et al.*, "Approximate analysis of limited-range wavelength conversion all-optical WDM networks," *Computer Commun.*, vol. 24, pp. 949–957, 2001.
[7] R. Barry, *et al.*, "Models of blocking probability in all-optical networks with and without wavelength changers," *IEEE J. Sel. Areas Commun.*, vol. 14, no. 5, June 1996.
[8] A. Patel, *et al.*, "Routing, wavelength assignment, and spectrum allocation in wavelength-convertible flexible optical WDM (WC-FWDM) networks," *2012 OFC/NFOEC.*
[9] S. Ross, *et al.*, *A First Course in Probability*. Prentice Hall, 2005.